Simple Machines:
Pulley

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ABSTRACT
In this activity, participants will see several pulley setups and, using Newton’s laws, calculate their respective mechanical advantages.

KEYWORDS
Newton’s laws, tension, weight, force, conservation of energy, pulleys, simple machine and mechanical advantage

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### Content Standards

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### History & Process Standards

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I. OBJECTIVES

Students will:
- learn about Newton’s laws.
- see several pulley setups and calculate their mechanical advantages.
- understand the concept of conservation of energy.
- understand why pulleys are helpful.

II. SAFETY

Some care should be taken with the large-scale demonstration. Inspect the door frame to insure it is sturdy enough to handle the load. Don’t allow anyone to lift a human with it, and insure no hands or feet are under the load when raising or lowering it. Also lifting the load directly could cause a back injury if not done carefully.

III. LEVEL TIME REQUIRED AND NUMBER OF PARTICIPANTS

A. LEVEL

Knowledge of simple multiplication and division is necessary to completely understand this module. Therefore, 3-12 grade students will benefit the most from its presentation, but younger children will still be able to operate and enjoy the large-scale demonstration.

B. TIME REQUIRED

The complete demonstration as presented takes about 15 minutes. The time can be lengthened by adding pulley configurations or by asking the students to design some of their own pulley systems. Additionally, approximately 15 minutes is required to set up the pulleys. Due to the amount of time required to set up the pulleys, it is important to not let them become disrupted by the students.

C. NUMBER OF PARTICIPANTS

3-15
IV. LIST OF MATERIALS

Pulleys
Masses
Striped String
Scissors
Large-scale Pulley System
Large Rock (40-50 lbs.)
Framed Doorway

V. INTRODUCTION

Pulleys are a type of simple machine designed to reverse the direction and reduce the amount of the force required to lift or move an object. Although pulleys were in use much earlier, Simon Stevin first enunciated their principles of action in about 1610.

A pulley system is simply one or more pulleys connected by rope or string. In order to analyze a pulley system, the knowledge of a few physical concepts is necessary. Mass is the measure of the amount of substance of an object. In the metric system, mass is measured in kilograms (kg). There is a platinum-iridium cylinder kept at the international Bureau of Weights and Measures in France that by definition has the mass of 1 kg. The mass of other objects can be found through the use of an equal arm balance. A force is defined as a push or pull in a particular direction. In analyzing pulley systems, the forces of interest are those caused by the acceleration of gravity acting on the masses in the system. The amount or magnitude of these forces is simply the mass of the object multiplied by the acceleration of gravity \( g = 9.8 \, \text{m}/(\text{s}^2) \) or symbolically

\[
F = mg
\]

As mentioned before, forces have a direction as well as a magnitude. Forces caused by gravity are directed downwards. Force is measured in Newtons (N). One Newton is the amount of force required to accelerate one kg at the rate of one meter per second squared.

Newton’s third law states that every force on a body is countered by an equal and opposite force. This applies to pulley systems as follows: if a weight is applied to a string an equal but opposite
force is generated in the string. The force in the string is
called tension. If the string were massless and nonaccelerating,
the tension would be constant throughout the string’s path. (In
this module, we will assume the string and pulleys have no mass
even though they do, because it makes the calculations easier and
the mass of the string and pulleys would only have a small effect
on the results.) Considering the tension in a string or rope to
be constant as it runs through the pulley system is the essential
principle in determining the effect of the system. The mechanical
advantage of the system is the ratio of the output force to the
input force of the system. The output force (F_{out}) of the system
is the weight of the object to be lifted while the input force is
the force required from the user to lift the object.

\[ \text{M.A.} = \frac{F_{out}}{F_{in}} \]

Newton’s first law states that a body persists in a state of rest
or constant velocity in a straight line unless an external force
acts upon it. By using Newton’s first law the mechanical
advantage of a pulley system can be determined experimentally.
The force required to lift a weight at a constant velocity is the
same as the force required to keep the system at rest. In other
words you can find the mechanical advantage of a pulley system by
applying a weight to the system and then determining the amount of
weight required on the free end of the string to keep the system
from moving. Through creative design, systems with various
mechanical advantages can be produced.

The mechanical advantage of a system can also be predicted
theoretically through the use of Newton’s third law and the use of
free-body diagrams. A free body diagram is simply a picture of a
body or element of the system indicating the forces acting on the
body. The forces are drawn as arrows pointing in the proper
directions at their points of application. When applied to pulley
systems, the body is usually a single pulley in the system and the
forces are tensions in the attached strings and the force of
gravity. By Newton’s third law, all of the forces acting on the
pulley must balance; the amount of the forces acting up must equal
the amount of the forces acting down. This along with the fact
that the tension in a string as it runs through the system is a
constant can be used to determine the amount of force required to
balance the system and thus its mechanical advantage.
For example, consider the pulley system shown in Figure 1. The load applied to the system is 98 N (F=10kg*9.8 m/s^2). Assuming a weight has been attached to the free-end of the string and the system is at rest the mechanical advantage of the system can be determined. The weight on the free end of the string would cause a tension, T, in the string equal to the weight. Next draw a

\[
2T = 98 \text{ N} \\
T = 49 \text{ N} \\
M = \frac{T}{g} \\
M = \frac{49}{9.8} \\
M = 5 \text{ kg}
\]
free-body diagram of the pulley to which the load is attached (see Fig. 2). Since the forces up and down must be equal we have:

\[
\text{Forces up} = \text{Forces down} \\
T + T = 98N \\
T = 49 N
\]

The tension of 49 N is the input force so the mechanical advantage is

\[
\text{M.A.} = 98 N : 49 N
\]

or simplifying

\[
\text{M.A.} = 2:1
\]

A few simplifying assumptions were made in these calculations: the pulley was considered massless, since otherwise another force equal to the weight of the pulley would have been included in the free-body diagram, the tension in the string was considered to be vertical on the right side of the pulley even though it isn’t quite vertical (see Figure 1), the mass of the string was neglected, and friction was neglected. Generally a reasonable approximation of the mechanical advantage of a pulley system can be obtained with these assumptions and they shall be used throughout this work.

The main limitation of pulley systems is the load is only raised a fraction of the distance of the length of string pulled through the system. One way to determine the distance the load will be raised for each length of string pulled through the system is through the use of the principle of conservation of energy. The conservation of energy principle required that the change in energy of the system is equal to the amount of work done on the system. In a pulley system, the change in energy from one state to another is equal to the weight of the load, \( W \), multiplied by the distance, \( d \), it is raised. The work done is the force, \( F \), applied to the free-end of the string multiplied by the distance, \( x \), over which it is applied:

\[
W \cdot d = F \cdot x
\]

or

\[
x/d = W/F = F_{out}/F_{in} = \text{M.A.}
\]
In other words this means the ratio of the length of string pulled through the system to the distance the load is raised is the same as the mechanical advantage of the system. For example if a system had a mechanical advantage of 4:1 and the user wished to raise the load 1 m then he/she would have to pull 4 meters of string through the system.

Another way to determine the ration of the string pulled through the system to the distance the load is raised is to consider the geometry of the system. In particular the length of the string in the system is constant. This “conservation of string” principle can best be illustrated by an example. For the system in Figure 1, if 2 cm of string is pulled out of the system at the free-end, this string must come from the two segments of string holding up the lower pulley. In particular 1 cm must come from each side, which means the load will only rise 1 cm. Note this is the same ratio as determined above and that this was obtained solely by the geometry of the system and in particular is independent of the weight of the pulleys or any friction in the system. The geometrical properties of the system also will determine if the system will function at all. It is possible to set up a system that will not operate because the “conservation of string” principle cannot be satisfied. A complete discussion of these limitations is beyond the scope of this presentation, but may be encountered if the students are allowed to design their own systems.

VI. PROCEDURE

Before the participants arrive, set up all of the pulley systems. Slid the hooks of the pulleys onto the dowel if the dowel does not have hooks on it as in the following picture. Otherwise, arrange the pulleys and string as shown in the following picture. The small-scale pulley systems require some patience and practice to assemble. They are much easier to assemble if a small load (such as a hangar) is applied to the system at the point the load will normally be applied. Initially just run the string through the pulleys without worrying about if it stays on the wheels or not. Once the string is routed through the system, carefully place it back on the pulley wheels.
small scale pulley system setups

The large-scale pulley system must be clamped to a doorframe and should always be stored with the rope strung through the system. Before hanging the pulley system, inspect the doorframe and decide if it is sturdy enough to hold a load of about 50 lbs. After inspecting the doorframe, simply clamp the system to the top of the frame and straighten out the ropes and chains. Once the ropes
and chains are straightened, adjust the chains to level the platform if necessary and place the rock on the platform.
B. EXECUTION

1) Begin with the single pulley loaded with 1200 g (see Fig. 1)

- Ask the participants how they could measure how much force would be required to lift the weight. If no suitable answer is given, convince them that a good way to determine the force required would be to see how much weight is necessary on the free end of the string to balance the system. Let them guess and try putting masses on the free end until arriving at
the correct answer of 1200 g. Tell them this is because the tension in the string is constant throughout its path and therefore the tension created by the 1200 g weight must be balanced by an identical weight.

- If desired, a distinction between mass and weight (a force) can be made at this point. However, it isn’t really necessary for these demonstrations and may complicate understanding of the basic concepts for younger participants. For more advanced participants, you can point out that this system could be used to determine the mass of an object if the mass of the load is known, since the effect of gravity cancels out.

- The mechanical advantage of the system is the ration of the weight lifted to the force applied and in this simple case is 1:1. The number of pulleys does not determine the mechanical advantage of the system. The mechanical advantage must be found by taking finding the tension of the string in the systems through the use of free body diagrams.

- Ask why such a pulley would be useful. If no one else does, point out that it reverses the direction of the force required to lift the load. This is very beneficial in overhead lifting and is used in most cranes.
2) Next demonstrate the two single pulley system with a 1200 g load (see Fig. 1b)

- Ask how much force will be required to lift the load. That is, ask how much weight or mass you need to attach to the free end of the string to balance the system. Try attaching different masses. Emphasize that the load attached to the end of the string creates a tension in the string wherever the string runs. Show them the large free-body diagram of the lower pulley. Since the sum of the forces in the vertical direction must be zero a load twice the weight of the tension in the string may be lifted.
\[ W = 2T \]

- This is equivalent to saying the System has a mechanical advantage of 2:1.

- Note that the result come from using the free-body diagram and not by counting the number of pulleys.

- Let the participants raise and lower the weight using the pulley system. Also have them lift the weight directly so they can feel that it is easier to lift with the aid of the pulleys.

- For advanced participants, note that for a given length of string pulled through the system the weight is only raised by half as much. This demonstrates the principle of conservation of energy. The principle of conservation of energy requires that the amount of work done on a system must equal the change in energy of the system. Work and energy are always in units of force multiplied by distance. The work done on the system in this case is the force applied to the free-end of the string multiplied by the distance it is moved. While the change in energy of the system is equal to the weight multiplied by the distance it is raised. So for this particular system

\[ Fd = Wh \]

or

\[ \frac{d}{h} = \frac{W}{F} = 2 \]
3) Next demonstrate the two triple-pulley system with a load of 1200 grams (see Fig. 1c).

- Hopefully, by this point the participant will quickly determine that 200 grams will be required on the free end of the string to balance the system. If not, show them the free
body diagram of the lower pulley. Again emphasize that for the system to be balanced that the forces in the vertical direction must be balanced.

\[ 6T = W \]

This is equivalent to saying the system has a mechanical advantage of 6:1

- **Note:** Balancing the system may require slightly more than the 200 gram estimate because until now we have neglected the weights of the pulleys. The triple-pulley is heavier than the single pulleys so some additional weight on the free-end of the string may be required to lift it. Friction in the system acts in the opposite direction of motion, however, and sometimes the friction in the pulley bearings will allow the 200 gram weight to be sufficient.

- Allow the participants to raise and lower the weight both directly and by using the system. The mechanical advantage will be much more apparent for this system. Also allow them to operate the system in reverse. Apply a load to the free end of the string and have them lift it by pulling on the bottom of the triple-pulley where the weight is normally applied.

- For advanced participants, you can again note that in this case because of energy conservation

\[ Fd = Wh \]

or

\[ \frac{d}{h} = \frac{W}{F} = 6 \]

- For advanced participants, if the striped string is used, it will also be evident that the different lengths of strings move at different rates. This is because if a certain length of string is pulled out of the free-end of the system the innermost length of string is only shortened by 1/6 as much. The second innermost string is also shortened by 1/6 as much, but the string from the innermost portion must pass through to get out of the system. Similarly the third innermost string also gets shorter by 1/6 the amount pulled out from the free-end but both of the amounts from the two inner strings must pass through this segment. If, for example, 6 inches of string is pulled out of the free-end of the system. The innermost segment of string is stationary while 1-inch
would move through the second innermost string, and two inches through the third innermost segment, etc.

4) **Next demonstrate the three single-pulley system with a load of 1200 grams** (see Fig. 1d).

![Diagram of a three-pulley system with a load of 1200 grams]

- When you ask them how much weight will be required on the free-end of the string, caution them to pay attention to the tension in the strings and not to count the number of pulleys. Note that there are two strings. From the free-body diagram of the first suspended pulley the tension in the second string must be twice the tension in the first string. The second string is then strung through another suspended pulley. The weight is attached to the bottom of this second pulley and from the free-body diagram of it the amount of the weight that can be lifted is doubled again for a total mechanical advantage of 4:1. So a force equal to \( \frac{1}{4} \) of the weight of the load will be required or in this case 300 grams will be required.
• Again this simple calculation may not be quite sufficient because the weight of the two suspended pulleys has been neglected.

• Allow the participants to operate the system by hand.

• For advanced participants from energy conservation

\[ Fd = Wh \]

or

\[ \frac{d}{h} = \frac{W}{F} = 4 \]
5) Finally, demonstrate the large-scale demonstration with a large rock on the platform.

- Let them carefully lift the load directly and then through the use of the pulley system.

- Ask the to guess the mechanical advantage of the system. It uses the same principles as the last small-scale demonstration and has a theoretical mechanical advantage of 16:1. You can show this with the aide of the free-body diagrams in Fig.5.
• Conservation of energy will be very evident in this system, since the load will only be raised by 1/16 as much as the amount of rope pulled through the free end.

C. CLEANUP

Leave the string on the pulley systems and neatly put them away to make it easier for the next person to run the demonstration. Always store the large scale pulley system with the string strung around the pulleys. Put the weights and hangars in their respective boxes. Discard any hopelessly knotted pieces of string.

VII. TROUBLE SHOOTING

It is hard to keep the string on the pulleys as you run it through the system. Putting a small weight, such as a hangar on the point where the load will latter be attached makes it easier. Also add a weight to the free-end of the string when you are done or tie the free-end to the eyelet in the base of the stand. Assistance from someone else and practice also cut down on the amount of time required to set up the demonstrations.

VIII. FREQUENTLY ASKED QUESTIONS

Where are pulleys used in real life? Pulleys are used in all sorts of ordinary items such as cranes, curtain pulls, garage doors and washing machines.

IX. REFERENCES

Physics: Algebra/Trig by Eugene Hecht

http://webphysics.iupui.edu/152/152Sp00/152Basics/newton/newton2.html