Angular momentum is always conserved. This activity gives a simple but amazing example of conservation of angular momentum.
Content Standards

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History & Process Standards

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I. OBJECTIVES
Students will:
- learn about rotational motion.
- observe an example of angular momentum being conserved.

II. SAFETY
- Don’t allow anyone to spin on the stand while standing up. It is easy to become dizzy.
- People with long hair should tie it back when doing the bicycle wheel demonstration: hair can get caught in the bicycle wheel spokes.
- Use the smaller wheel for people with shorter arms. The larger wheel is appropriate only for adults.
- Be careful that the volunteers don’t touch the inside of their arms with the wheel while it is spinning: it will cause friction burns.
- Be ready to catch people if they stand up suddenly and get dizzy.

III. LEVEL, TIME REQUIRED AND NUMBER OF PARTICIPANTS

LEVEL
The activity is appropriate for all levels, though very young participants may not be able to hold the bicycle wheel by him or her self.

TIME REQUIRED
The activity takes 5-10 minutes to present, depending on the complexity of the explanation. Letting each person do the activity him or herself can take longer.

NUMBER OF PARTICIPANTS
Groups of 6 - 10 work well.
IV. LIST OF MATERIALS

angular momentum turntable (blue, heavy)
large and small bicycle wheels
rope
leather palmed glove

V. INTRODUCTION

If you think about the wheel of a car moving down a road, you can divide its motion into two parts: rotation and translation. The wheel is turning around and, at the same time, the center of the wheel is moving in a straight line. People are generally more familiar with linear motion (translation) than they are with rotation, so explaining this demonstration from the perspective of linear motion is often helpful. Linear momentum, symbolized by \( p \), is defined as the product of the mass of an object, \( m \), and its velocity, \( v \):

\[
p = mv
\]

Both momentum, \( p \), and velocity, \( v \), are vectors -- they have not only a quantity, but also a direction associated with them. If an object is moving toward the right, its momentum vector will point toward the right. If the object is moving toward the left, its momentum vector will point toward the left. If you are driving a 4000 kg car that is moving 100 km/hr from Lincoln to Omaha, the momentum of the car is \((4000 \text{ kg})(100 \text{ km/hr East}) = 400000 \text{ kgkm/hr East}\).

The angular velocity is analogous to the linear velocity. Angular velocity, \( \omega \), is proportional to the number of rotations you make per second. For example, two ants are sitting on a record that is going round and round in a phonograph. One ant is sitting on the edge of the record and the other is sitting near the center. Both ants go around in a circle in the same period of time. But, the one on the edge is making bigger circles, and thus is actually traveling farther. So, both ants have the same angular velocity (it takes them each the same amount of time to go around the center of the phonograph) but at the same time they have different linear velocities (one aunt makes a bigger circle in the same amount of time that the other makes a small circle).

To understand the math involved: a person sitting on a stool rotating twenty times per minute has an angular velocity, \( \omega \), of

\[
\omega = (2\pi \text{ radians/rotation})(20 \text{ rotations/minute})(1\text{ minute/60 seconds}) = (2/3)\pi \text{ radians/second}
\]

\[
\omega = 2\pi \frac{20}{60 \text{ seconds}} = 21 \frac{1}{\text{ second}}
\]
By multiplying by $2\pi$, we turn rotations per second into radians per second. Scientists usually work in radians instead of degrees or rotations. $2\pi$ radians is equivalent to 360 degrees.

The mass we use in calculating linear momentum is replaced in rotational momentum by something called the moment of inertia, $I$. Just as mass measures how easy or hard it is to get something moving, the moment of inertia measures how easy or hard it is to get something rotating. Formally,

$$I = \sum m_i r_i^2$$

This says that the further the mass of an object is located from its axis of rotation, the harder it is to get the object rotating.

So, for example, let’s compare a cylinder to a hoop, both of which have the same mass and the same radius and are rotated about an axis passing through their center, perpendicular to the disk. The moment of inertia of the cylinder is:

$$I = \frac{1}{2} MR^2$$

and that of the hoop is:

$$I = MR^2$$

This tells us that, because all of the mass in the hoop is concentrated at the edges of the hoop, it is harder to get the hoop rotating or to stop it from rotating. The mass of the cylinder, in comparison, is distributed, making it easier to get the object rotating.

Just as you can calculate a kinetic energy for an object moving in a straight line ($KE = \frac{1}{2} mv^2$), we can also calculate the kinetic energy of a rotating object with moment of inertia $I$ and angular velocity $\omega$ which is given by:
When an object rotates, there is a measure of momentum analogous to linear momentum, which we call angular momentum, \( L \). The angular momentum is given by:

\[
L = I \omega
\]

Angular momentum is a quantity that is conserved: that is, it is neither gained nor lost. We write this as:

\[
L = \text{constant}
\]

If \( L \) is constant, changing \( I \) means that \( \omega \) must also change. If the moment of inertia increases, the angular frequency, \( \omega \), must decrease. This is consistent with the fact that it is harder to rotate an object with a higher moment of inertia and, given the same energy, the object must rotate more slowly.

**VI. PROCEDURE**

**A. SET UP**
Use the stanchions to cordon off an area about 6 square feet.

**B. Execution**

**The Bicycle Wheel**
A bicycle wheel is another good object for studying rotation. Have your volunteer sit on the stool. Using the leather glove to protect your hand, start the bicycle wheel spinning. Get the wheel spinning fast, with the handles parallel to the ground. Hand the wheel to the volunteer on the stool. The volunteer is not rotating, therefore, your volunteer has no angular momentum. The bicycle wheel has a certain amount of angular momentum because it is rotating. The total angular momentum is now:

\[
L_{\text{total}} = L_{\text{person}} + L_{\text{wheel}}
\]

\[
L_{\text{total}} = 0 + L_{\text{wheel}}
\]

Ask the volunteer to turn over the bicycle wheel. The angular momentum changes from \( L_{\text{wheel}} \) to \(-L_{\text{wheel}}\). Again, angular momentum must be conserved.
\[ L_{\text{total}} = (L_{\text{person}})_{\text{before}} + (L_{\text{wheel}})_{\text{before}} = (L_{\text{person}})_{\text{after}} + (L_{\text{wheel}})_{\text{after}} \]

We have that:

\[
\begin{align*}
(L_{\text{person}})_{\text{before}} &= 0 \\
(L_{\text{wheel}})_{\text{before}} &= +L \\
(L_{\text{wheel}})_{\text{after}} &= -L
\end{align*}
\]

From conservation,

\[
\begin{align*}
(L_{\text{person}})_{\text{before}} + (L_{\text{wheel}})_{\text{before}} &= (L_{\text{person}})_{\text{after}} + (L_{\text{wheel}})_{\text{after}} \\
0 + L &= (L_{\text{person}})_{\text{after}} - L \\
(L_{\text{person}})_{\text{after}} &= 2L
\end{align*}
\]

Thus the person must acquire an angular velocity. The total angular momentum after all is still +L. Note that the signs on the person’s angular momentum and the bicycle wheel’s angular momentum are opposite: this means that the person and the wheel must be rotating in opposite directions. If the person turns the wheel over to its original position, he/she should stop rotating: now the wheel has returned to its original angular momentum of +L, so the rotation of the person is no longer necessary to maintain conservation of angular momentum.

One thing we haven’t discussed is the conditions under which angular momentum is conserved. The requirement is that no external torque be applied to the system. So, for example, if the person (holding the wheel) and the wheel are both still (total angular momentum zero) and then you start the wheel rotating, the system of the person and the wheel will not have zero total angular momentum, because you exerted a torque on the wheel. One way to demonstrate this is the following:

Have the person sit on the stool and start the wheel spinning. Hand it to them and have them turn it over, which will cause them to start rotating. When they spin around to where you are again, take the wheel from them as they continue rotating. Turn the wheel over and hand it to the person as they come around again. Each time, you are adding angular momentum to the system -- actually handling some angular momentum to the spinning volunteer.
Precession

Tie a string to one of the bicycle wheel handles. Hold the bicycle wheel by both handles so that the handles are parallel to the ground. Then, while still holding the wheel in the same position with one hand, grab a hold of the end of the string with the other. Now release the handle so the wheel is held by the string only. Notice that the bicycle wheel drops so that the handles are perpendicular to the ground.

Start the bicycle wheel spinning and then repeat the previous precession steps. Notice that this time, the wheel does not drop when held by the string only. The wheel spins around its axis and also revolves around the string.

Why does this happen? It is a phenomenon called precession. It is easier to understand the precession if we think of a small piece of the bicycle wheel instead of the entire wheel. Gravity is exerting a downward force on a piece of the spinning wheel. But at the same time that piece is being moved in another direction because wheel is spinning. So each piece of the wheel is not just being pushed down, it is being pushed in two directions. The combined downwards push of gravity and the movement of the wheel causes the wheel to rotate around the string instead of moving down.
VII. FREQUENTLY ASKED QUESTIONS

Why can’t I stand up?
It’s not safe -- you get a better spin by sitting down because you are more evenly situated on the chair.

VIII. TROUBLE SHOOTING

IX. HANDOUT MASTERS

none

X. REFERENCES

Physics: algebra/trig by Eugene Hecht
http://library.thinkquest.org/3042/angular.html